

## *Chapter 14*

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### CONDUCTION THROUGH A GAS AT LOW CURRENTS

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Gas tubes are a class of electron tubes in which conduction takes place through a partially ionized gas between the electrodes. Some types of gas tubes include: rectifiers, voltage-reference tubes, voltage-regulator tubes, counting tubes, tubes used as switching elements, storage elements, indicator lamps, and sources of illumination. Gas lasers also make use of a gas discharge. For each type of service the design of the tube is different.

As an introduction to the subject of gas tubes we shall discuss in the present chapter some of the mechanisms involved in gas discharges at low currents between unheated electrodes. The range of currents we shall consider extends from perhaps a few electrons per second to a few microamperes. In the two chapters that follow we shall consider discharges at higher currents, and we shall describe a number of specific devices.

Both ions and electrons contribute to the conduction of current in a gas tube, their motions being relatively complex because of frequent collisions with other particles in the interelectrode space. Since many of the processes taking place in a discharge depend on particle collisions, and since collision processes are inherently statistical in nature, it is usual to describe the discharge phenomena in terms of average particle behavior rather than individual particle histories. In regions of uniform electric field intensity a statistical description of the particle behavior in terms of the average particle density, the particle drift velocity, and the distribution of particle energies can sometimes be given. Some data are also available on the excitation and ionization caused by electrons moving through a gas under the influence of a uniform electric field. However, discharges often involve regions of rapidly changing fields, and when this is the case, much less is generally known about the particle behavior.

In addition to ions and electrons, photons and excited atoms called metastables often play an important part in the discharge.

Many gas tubes have cathodes that are not heated and hence are called cold-cathode tubes. When a discharge is established in such a tube, an electron leaving the cathode gives rise to a number of discharge products, ions, metastables, and photons, as well as additional electrons. Many of the ions are drawn to the cathode by the electric field, and some of the metastables and photons reach the cathode. Each has a certain probability, less than one, of releasing another electron. At currents at which cold-cathode tubes are normally operated, the current drawn by the tube must be limited by a series resistance, and the voltage across the tube adjusts itself so that the average number of discharge products generated by an electron leaving the cathode is sufficient to cause the release of one more electron from the cathode. Electron emission is thus supported by the discharge itself, and the discharge is said to be self-sustaining.

In order to establish a self-sustained discharge in a cold-cathode tube, the voltage applied to the tube must be raised above a value known as the breakdown voltage. Over a range of currents covering several orders of magnitude up to about 10 microamps, the anode-to-cathode voltage of the self-sustained discharge is equal to the breakdown voltage, the condition for breakdown being the same as that for a self-sustained discharge at low currents. At higher currents, above about 10 microamps, the sustaining voltage falls below the breakdown voltage, and a visible glow can be seen in the interelectrode space. Above 200 microamps the glow is fairly intense, and the discharge is said to be a glow discharge. Most cold-cathode tubes are normally operated in the glow-discharge region.

It should be noted at this point that provided the cathode area is sufficiently large, the discharge current for a given applied voltage is nearly independent of the cathode area at low discharge currents. This occurs because a portion of the cathode surface will always have slightly more favorable surface conditions for electron emission than the rest of the cathode, and the discharge tends to start opposite this part of the cathode. Once the discharge is initiated, the ion bombardment of the cathode in the region of the discharge tends to keep this part of the cathode somewhat more free of absorbed contaminant gases, with the result that it remains a better electron emitter. Thus for discharge currents below 200 microamps, the remainder of the cathode surface plays very little part in the discharge, provided the cathode area is sufficiently large. It is therefore customary to describe the discharge at low discharge currents in terms of the total discharge current rather than the discharge current density.

Voltage-reference tubes, voltage-regulator tubes, counting tubes, and

neon indicator lamps are all cold-cathode devices. Some switching tubes and storage elements are also cold-cathode devices.

The gas fillings of cold-cathode tubes are usually noble gases, since these do not interact with the electrode material, and more stable electrical characteristics can be obtained. Often neon, or argon, or a mixture of neon and argon are used. One material frequently used for the cathodes of voltage-reference and voltage-regulator tubes is molybdenum, since it can be processed to give a particularly clean surface. Recently niobium has also been shown to give very stable cathode surfaces when used with the noble gases. Cold-cathode tubes used in switching applications often have oxide-coated nickel cathodes and are filled with a mixture of neon and argon.

In the first sections of this chapter we shall describe the motions of ions, electrons, and metastables through the noble gases. Later we shall discuss the low-current discharge in neon gas between planar molybdenum electrodes. In Chapter 15 we shall discuss the glow discharge and some specific cold-cathode tubes. In Chapter 16 we shall describe a quite different form of discharge known as the hot-cathode discharge.

In this chapter and in the remaining chapters we shall consider only dc discharges in which both the electrode voltages and currents are constant with time. Consequently we shall omit the subscript "o" which was used in previous chapters to designate dc quantities.

#### 14.1 The Kinetic Nature of a Gas

The molecules of the noble gases are single atoms which for many purposes behave like hard elastic spheres. They travel at average velocities of the same order of magnitude as the speed of sound in the gas and experience frequent elastic collisions with each other. At pressures normally used in cold-cathode tubes the average distance traveled in the interval between collisions is very large compared with the molecular diameter and small compared with typical electrode dimensions and spacings.

The density of molecules in a gas at 0°C is equal to  $3.54 \times 10^{16} p$  molecules per cubic centimeter, where  $p$  is the pressure in millimeters of Hg. Cold-cathode tubes are often filled to pressures of the order of 10 to 100 mm of Hg. (1 mm of Hg is equivalent to 1/760 of atmospheric pressure or 133 newtons/meter<sup>2</sup>). At  $p = 50$  mm of Hg there is an average volume of space per molecule equal to the volume of a cube of side 83 angstroms (one angstrom =  $10^{-8}$  cm). In this chapter and in subsequent chapters we shall use  $p$  to designate pressure in millimeters of Hg and  $P$  to designate pressure in mks units.

Approximate molecular diameters<sup>1</sup> for the noble gases neon and argon

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<sup>1</sup>*Viscosity Diameters, Landolt-Bornstein Tables, Julius Springer, Berlin, 1950, Vol. 1, Part 1, p. 325.*

are 2.67 angstroms and 3.76 angstroms, respectively. Equation (14.1-8) below gives the mean distance traveled by a molecule between collisions (mean free path) as  $1/(\sqrt{2}n\pi d^2)$ , where  $n$  is the number of molecules per unit volume, and  $d$  is the molecular diameter. Using the approximate molecular diameters given above we find that at 50 mm of Hg and 0°C, the mean free path of a neon molecule in neon is 18,000 angstroms, and the mean free path of an argon molecule in argon is 9000 angstroms.

At 0°C the average velocity of neon molecules is 538 meters/sec, and the average velocity of argon molecules is 381 meters/sec. We shall find shortly that the average kinetic energy of the molecules is directly proportional to the absolute temperature of the gas.

From statistical mechanics we learn how the molecular velocities are distributed. The law that expresses this distribution is known as the Maxwell-Boltzmann distribution law.<sup>2</sup> It states that the probability  $M$  that a molecule has velocity  $u$  with components in the range  $u_x$  to  $u_x + du_x$ ,  $u_y$  to  $u_y + du_y$ ,  $u_z$  to  $u_z + du_z$  is given by

$$M = (2\pi kT/m)^{-3/2} e^{-mu^2/2kT} du_x du_y du_z \quad (14.1-1)$$

where  $u^2 = u_x^2 + u_y^2 + u_z^2$ ,  $k$  = Boltzmann's constant,  $T$  is the absolute temperature of the gas, and  $m$  is the mass of the molecules.

It is convenient to imagine a velocity space in which three rectangular coordinate axes measure the velocity components  $u_x$ ,  $u_y$ , and  $u_z$ . Each molecule in a given enclosed volume can at any time be represented by a point in such a space, and from Equation (14.1-1) we see that the fractional density of points in the space, or the number of points per unit volume di-

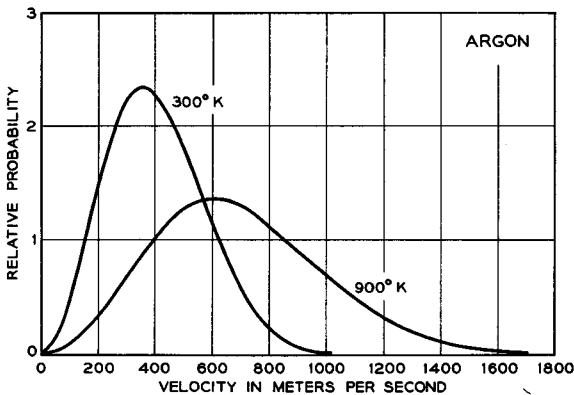


FIG. 14.1-1 Velocity distributions of argon molecules for gas temperatures of 300°K and 900°K.

<sup>2</sup>Reference 14.1, p. 90; Reference 2, p. 52.

vided by the total number of points, is equal to  $(2\pi kT/m)^{-3/2} \epsilon^{-mu^2/2kT}$ . It follows that the fraction of the total points lying in a spherical shell of radius  $u$  and thickness  $du$ , and hence the fraction of the total number of molecules having velocities in the range  $u$  to  $u + du$ , is given by

$$f(u)du = (2\pi kT/m)^{-3/2} \epsilon^{-mu^2/2kT} 4\pi u^2 du \quad (14.1-2)$$

The velocity distribution given by this expression for  $T = 300^\circ\text{K}$  and  $T = 900^\circ\text{K}$  is plotted in Figure 14.1-1 for argon molecules.

Using Equation (14.1-2), the average kinetic energy of the molecules is easily shown to be

$$\begin{aligned} (\text{k.e.})_{\text{avg}} &= 2\pi m(2\pi kT/m)^{-3/2} \int_0^\infty u^4 \epsilon^{-mu^2/2kT} du \\ &= (3/2)kT \end{aligned} \quad (14.1-3)$$

The average kinetic energy of the molecules is thus directly proportional to the absolute temperature of the gas and is independent of the mass of the molecules and hence of the kind of gas.

In a similar manner, the average molecular velocity can be shown to be given by

$$u_{\text{avg}} = 4(kT/2\pi m)^{1/2} = (8kT/\pi m)^{1/2} \quad (14.1-4)$$

Next let us determine the pressure on the walls of the containing vessel. From Equation (14.1-1) we can determine the fraction of the total number of molecules with velocity in the range  $u_x$  to  $u_x + du_x$ . This fraction is given by

$$\begin{aligned} F(u_x)du_x &= (2\pi kT/m)^{-3/2} \left[ \int_{-\infty}^\infty \int_{-\infty}^\infty \epsilon^{-mu^2/2kT} du_y du_z \right] du_x \\ &= (2\pi kT/m)^{-1/2} \epsilon^{-mu_x^2/2kT} du_x \end{aligned} \quad (14.1-5)$$

The number of molecules striking unit area of surface normal to the  $x$  direction per unit time with velocities in the range  $u_x$  to  $u_x + du_x$  is  $u_x nF(u_x)du_x$ , where  $n$  is the number of molecules per unit volume. If the molecules are reflected from the wall so that their  $x$  component of velocity is reversed, each molecule suffers a change in momentum  $2mu_x$  upon collision with the wall. The pressure is therefore given by

$$P = 2mn(2\pi kT/m)^{-1/2} \int_0^\infty u_x^2 \epsilon^{-mu_x^2/2kT} du_x = nkT \quad (14.1-6)$$

(Actually collisions between the gas molecules and the walls are not always elastic and, in fact, sometimes result in the molecule "sticking" to the surface. However, in a steady-state condition in which the temperature of the walls equals the temperature of the gas, the current of molecules

arriving at the surface equals the current leaving, and the average energy of the incident molecules equals that of the departing molecules. Hence, for the purpose of determining the pressure on the walls, it is possible to treat the interactions at the surface as though only elastic reflections are taking place.)

Rearranging Equation (14.1-6), we find that  $n$ , the number of molecules per unit volume, is equal to  $P/kT$ . Since  $P/kT$  is independent of the mass of the molecules, we can conclude that for the same conditions of temperature and pressure all gases have the same number of molecules per unit volume. This result is known as Avogadro's Hypothesis.

If we let  $N$  equal the number of molecules in one gram molecular weight of gas and let  $V$  be the volume occupied by this amount of gas, then

$$P = (N/V)kT$$

or

$$PV = RT \tag{14.1-7}$$

where  $R = Nk$  is a universal constant for one mole of gas equal to 8.31 joules/°K. Equation (14.1-7) is known as the equation of state for a perfect gas.

As a final point let us consider the average distance traveled by molecules between collisions. We shall first assume that only one molecule is moving and that the rest are stationary and randomly distributed throughout space with an average density of  $n$  molecules per cubic meter. If the molecules have an effective diameter  $d$  for the collision process, a collision takes place whenever the center of the moving molecule passes within  $d$  meters of the center of a stationary molecule. The effective volume swept out by the moving molecule per unit time is therefore  $u\pi d^2$  cubic meters per second, where  $u$  is the velocity of the moving molecule. The average number of collisions made by the moving molecule per unit time is  $nu\pi d^2$ , and the mean distance traveled between collisions (mean free path) is  $u/nu\pi d^2 = 1/n\pi d^2$ .

This derivation of the mean free path is only an approximate one, however, for we have neglected the effect of the motion of the other molecules. When this additional motion is taken into account, it can be shown that the mean free path  $L$  is given by<sup>3</sup>

$$L = \frac{1}{\sqrt{2} n\pi d^2} \tag{14.1-8}$$

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<sup>3</sup>Reference 14.1, p. 110.

**14.2 Diffusion and Mobility**

(a) *Diffusion*

When two dissimilar gases are mixed in such a manner that concentration gradients exist within the mixture, the concentration gradients tend to become diminished by a process known as diffusion. As an example of this process we shall consider what happens when the gases CO and N<sub>2</sub> are mixed. We shall assume that the gases are initially separated by a partition as in the apparatus shown in Figure 14.2-1 and that the pressure on both

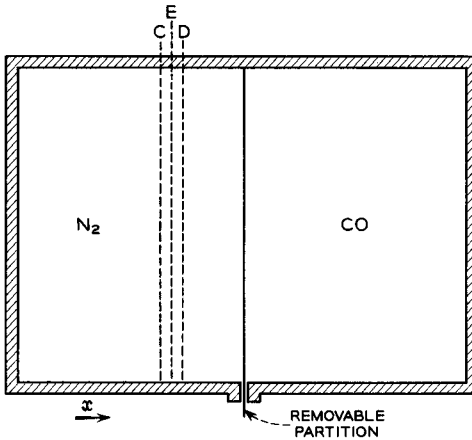


FIG. 14.2-1 A chamber with removable partition.

sides of the partition is the same. The gases CO and N<sub>2</sub> have been selected because their molecules have the same mass and travel with the same average velocity. This means that each diffuses into the other at approximately the same rate, which considerably simplifies the problem.

When the partition is removed, there is a net flow of N<sub>2</sub> molecules to the right and a flow of CO molecules to the left. (We shall assume that there is no turbulence or convection within the gas.) The flow results from the thermal motions of the molecules and continues until the partial pressure of the two gases is uniform throughout the entire chamber.

Diffusion processes are always characterized by a net flow of the diffusing particles away from regions of high concentration. To explain this, let us first calculate the current density of N<sub>2</sub> molecules crossing a plane within the gas. Using Equation (14.1-5), we find that the current of N<sub>2</sub> molecules crossing unit area within the gas is given by

$$n \int_0^{\infty} u_x F(u_x) du_x = n(kT/2\pi m)^{1/2} = nu_{avg}/4 \tag{14.2-1}$$

where  $n$  is the density of  $N_2$  molecules and  $u_{\text{avg}}$  is the average molecular velocity given by Equation (14.1-4). Hence the current of  $N_2$  molecules crossing a plane within the gas is proportional to the density of  $N_2$  molecules at the plane and to the average molecular velocity.

Next let us consider the flow of  $N_2$  molecules across planes  $C$  and  $D$  in Figure 14.2-1. If the partial pressure of  $N_2$  molecules at plane  $C$  is greater than that at plane  $D$ , the current of  $N_2$  molecules crossing plane  $C$  in the direction from left to right will be greater than that crossing plane  $D$  from right to left. As a result, there will be a net current of  $N_2$  molecules flowing in the direction from left to right across plane  $E$  midway between planes  $C$  and  $D$ .

As a very rough approximation let us assume that the total current of  $N_2$  molecules crossing plane  $E$  from left to right is characteristic of the density of  $N_2$  molecules  $L$  units to the left of plane  $E$ , where  $L$  is the mean free path. Similarly, let us assume that the total current of  $N_2$  molecules crossing plane  $E$  from right to left is characteristic of the density of  $N_2$  molecules  $L$  units to the right of plane  $E$ . Then the net current density  $J$  of  $N_2$  molecules crossing plane  $E$  is given by

$$\begin{aligned} J &= \frac{u_{\text{avg}}}{4} [n_{(x_o-L)} - n_{(x_o+L)}] \\ &= \frac{u_{\text{avg}}}{4} \left( n_{x_o} - L \frac{dn}{dx} - n_{x_o} - L \frac{dn}{dx} \right) \\ &= -\frac{Lu_{\text{avg}}}{2} \frac{dn}{dx} \end{aligned} \quad (14.2-2)$$

where the  $x$  direction is taken normal to the planes  $C$ ,  $E$ , and  $D$  and the coordinate of plane  $E$  is given by  $x = x_o$ . Although this equation is only approximate, it is apparent that we can express the current density  $J$  in the form

$$J = -D \frac{dn}{dx} \quad (14.2-3)$$

where  $D$  is a constant of dimensions length<sup>2</sup>/time. The constant  $D$  is known as the diffusion coefficient. More generally, diffusion takes place in three dimensions, and we can write that

$$\mathbf{J} = -D \nabla n \quad (14.2-4)$$

Since the  $N_2$  molecules are conserved, we can invoke an equation of continuity on them, namely that

$$\nabla \cdot \mathbf{J} = -\frac{\partial n}{\partial t} \quad (14.2-5)$$



Combining Equations (14.2-4) and (14.2-5), we obtain

$$D\nabla^2 n = \frac{\partial n}{\partial t} \quad (14.2-6)$$

This equation is known as the diffusion equation. It is of similar form to that which applies to the flow of heat in conducting bodies, and a mental picture of a particular diffusion process can often be obtained in terms of heat flow in conducting bodies of similar geometry. (In the case of heat flow,  $n$  would be replaced by temperature or heat energy, and  $J$  would represent flow of heat energy.)

The diffusion coefficient  $D$  is inversely proportional to the total gas pressure, and experiment shows that it is not in general critically dependent on the relative composition of the two gases. An approximate expression for  $D$  is given by Equations (14.2-2) and (14.2-3), namely  $D = Lu_{\text{avg}}/2$ . A more accurate expression that takes into account the distribution of molecular velocities is given by<sup>4</sup>

$$D = \frac{3\pi\sqrt{2}}{32} Lu_{\text{avg}} \sqrt{\frac{m+M}{M}} \quad (14.2-7)$$

where it is assumed that molecules of mass  $m$  and average velocity  $u_{\text{avg}}$  are diffusing into a gas of molecular mass  $M$ , and where the molecules are assumed to behave like hard elastic spheres upon collision with one another. The diffusion coefficient for electrons diffusing through a gas is given by<sup>5</sup>  $D = Lu_{\text{avg}}/3$ , where  $u_{\text{avg}}$  is the average electron velocity, and  $L$  is the electron mean free path for collisions with the gas molecules.

Diffusion takes place in a number of physical processes which are characterized by random particle motion and mean free paths. When ions, metastable atoms, or electrons are present in a gas, there is a tendency for nonuniformities in the particle concentration to be evened out as a result of diffusion.

If a boundary of the chamber is also a sink for the diffusing particles, as is the case when metastable atoms or ions diffuse through a gas to a metal surface, the boundary condition  $n = 0$  applies at the surface.

It is sometimes useful to regard diffusing molecules as being driven by their partial pressure gradient. To understand this view of the process let us first consider the diffusion of a gas through a porous carbon block as illustrated in Figure 14.2-2. Here again the gas penetrates the block because of the random motion of the gas molecules. If the block has unit cross

<sup>4</sup>Reference 14.1, Sections 109, 267.

<sup>5</sup>See Reference 14.1, Section 269, for a derivation of the electron mobility. Using Equations (14.1-4) and (14.2-12) of the present text, the above expression for the electron diffusion coefficient can be obtained from the electron mobility.

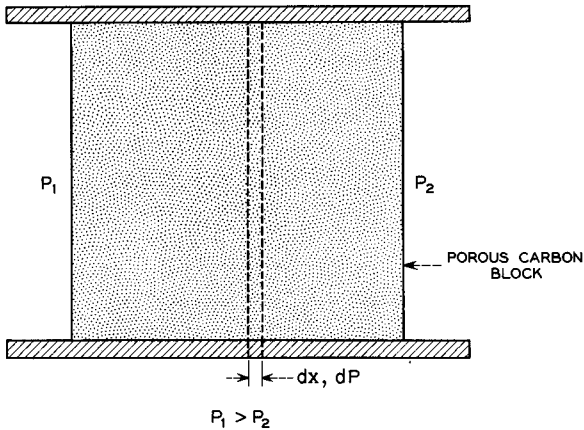


FIG. 14.2-2 A porous carbon block. The left-hand side of the block is in contact with a gas at pressure  $P_1$ , and the right-hand side is in contact with the same gas at pressure  $P_2$ .

section, the gradient of partial pressure of the gas  $dP/dx$  in the direction of diffusion is equal to the resistive force per unit length that the block offers to the flow of gas. The partial pressure difference  $dP$  over the length  $dx$ , effectively exerts an average force  $dP/ndx$  on each gas molecule, where  $n$  is the density of gas molecules in the slab of thickness  $dx$ .

Similarly in the diffusion of  $N_2$  into CO the gradient of the partial pressure of the  $N_2$  molecules equals the resistive force per unit area and per unit length in the direction of diffusion offered by the CO molecules. In this case the resistance offered by the CO molecules arises from the partial pressure gradient of the CO molecules and is directed oppositely to the partial pressure gradient of the  $N_2$  molecules. We shall find this view of the diffusion process useful in the discussion of the relationship between diffusion and mobility given below.

### (b) Mobility

Next let us consider the motion of positively charged atomic ions of the noble gases through their parent gases. We shall assume that the density of the ions is sufficiently small that the field resulting from the charge of an ion has negligible effect on the motion of the other ions. If no external fields are applied, the ions behave much like their parent gas molecules. They have essentially the same mass as the parent molecules and exhibit much the same velocity distribution. (However, their cross section for collision is probably somewhat larger than that of the parent molecules.) Any net motion of the ions in this case results from diffusion.

If, on the other hand, there exists a uniform electric field intensity  $E$  in the region, the ions acquire a drift velocity in the direction of the field. If the ions are singly charged, the electric field acts on them with a force  $eE$  newtons at all times and gives them an acceleration  $eE/m$  in the direction of the field, where  $m$  is the mass of an ion. In the steady-state condition the energy gained by the ions in drifting under the influence of the field is transmitted to the gas molecules in elastic collisions, so that the average drift velocity of the ions is a function only of the magnitude of the electric field intensity and not of how long the ions have drifted under the influence of the field. Consequently at pressures normally used in cold-cathode tubes the power expended by the external source applying the field appears largely as heat energy of the neutral gas molecules rather than as kinetic energy of the ions.

For weak electric fields the average drift velocity  $\bar{u}$  of the ions is linearly proportional to the applied electric field intensity  $E$  and can be expressed as

$$\bar{u} = \mu E \quad (14.2-8)$$

where  $\mu$  is a constant known as the mobility of the ions.

The resistance which ions encounter in drifting through a gas under the influence of a weak electric field is essentially the same as the resistance encountered by ions diffusing through the gas. It is reasonable therefore to expect the existence of a direct correlation between the diffusion coefficient  $D$  and the mobility  $\mu$ .

In the diffusion of ions there is effectively a force  $dP/ndx$  acting on each ion causing it to drift in the direction of the partial pressure gradient, the  $x$  direction, where  $P$  measures the partial pressure of the ions, and  $n$  measures the ion density. It is convenient to associate a velocity  $u'$  with the diffusion process such that  $J = nu' = -Ddn/dx$ , where  $J$  is the number of ions crossing unit area normal to the partial pressure gradient per second, and  $n$  is the ion density. The velocity  $u'$  can be expressed in the form

$$u' = -\frac{D}{n} \frac{dn}{dx} = -\frac{D}{P} \frac{dP}{dx} \quad (14.2-9)$$

It follows that the force  $dP/ndx$  acting on each ion as a result of the partial pressure gradient is given by

$$\left| \frac{dP}{ndx} \right| = \frac{Pu'}{nD} \quad (14.2-10)$$

If, on the other hand, there is no gradient in the concentration of ions, but an electric field  $E$  exists in the region causing the ions to drift with a velocity  $\bar{u}$  equal to  $u'$ , then the force  $eE$  must equal  $dP/ndx$ , or

$$eE = \frac{e\bar{u}}{\mu} = \frac{eu'}{\mu} = \frac{Pu'}{nD} \quad (14.2-11)$$

Hence<sup>6</sup>

$$\frac{D}{\mu} = \frac{P}{en} = \frac{kT}{e} \quad (14.2-12)$$

where the relationship  $P = nkT$  given by Equation (14.1-6) is assumed to apply to the ions diffusing through their parent gas. This equation gives the desired relation between the mobility and the diffusion constant.

If it is assumed that collisions between the ions and the gas molecules are elastic, Equations (14.2-7) and (14.2-12) can be combined to give an expression for  $\mu$ , namely that

$$\mu = \frac{e}{kT} \frac{3\pi\sqrt{2}}{32} Lu_{avg} \sqrt{\frac{m+M}{M}} \quad (14.2-13)$$

where  $m$  is the mass of the ions and  $M$  is the mass of the gas molecules.

### 14.3 Motion of Ions and Electrons through a Gas under the Influence of an Electric Field — Experimental Results

In this section we shall first present some results of measurements of the drift velocities of noble-gas ions through their parent gases when a uniform electric field is applied. Later we shall discuss the motion of free electrons through a noble gas under the influence of a uniform electric field.

Figure 14.3-1 shows some measured values<sup>7</sup> of the drift velocities of noble-gas ions through their parent gases as a function of  $E/p$ , the ratio of the electric field intensity to gas pressure. The quantity  $E/p$  is an independent parameter in a number of gas discharge phenomena since at constant  $E/p$  the average energy which an ion or electron gains in the interval between collisions is a constant. Furthermore, since the ion or electron loses kinetic energy with each collision, it must reach an equilibrium kinetic energy which depends only on the energy gained between collisions. It is customary in gas-discharge work to measure  $E$  in volts/per centimeter and  $p$  in millimeters of mercury rather than in mks units. This convention will be adhered to in the discussion that follows.

The log-log plot in Figure 14.3-1 shows a slope of one at low values of  $E/p$ , indicating that  $\bar{u}$  is proportional to  $E/p$  and hence to  $E$  in agreement with Equation (14.2-8). On the other hand, at high values of  $E/p$  the curves have a slope of 1/2, indicating that  $\bar{u}$  is proportional to  $\sqrt{E/p}$ . An analysis of the problem<sup>8</sup> which takes into account the nature of the ion-atom forces is able to predict this variation of  $\bar{u}$  with  $E/p$ .

<sup>6</sup>When the ion velocity distribution is not Maxwellian, this relationship is not strictly true.

<sup>7</sup>References 14.3 and 14.4.

<sup>8</sup>Reference 14.5.

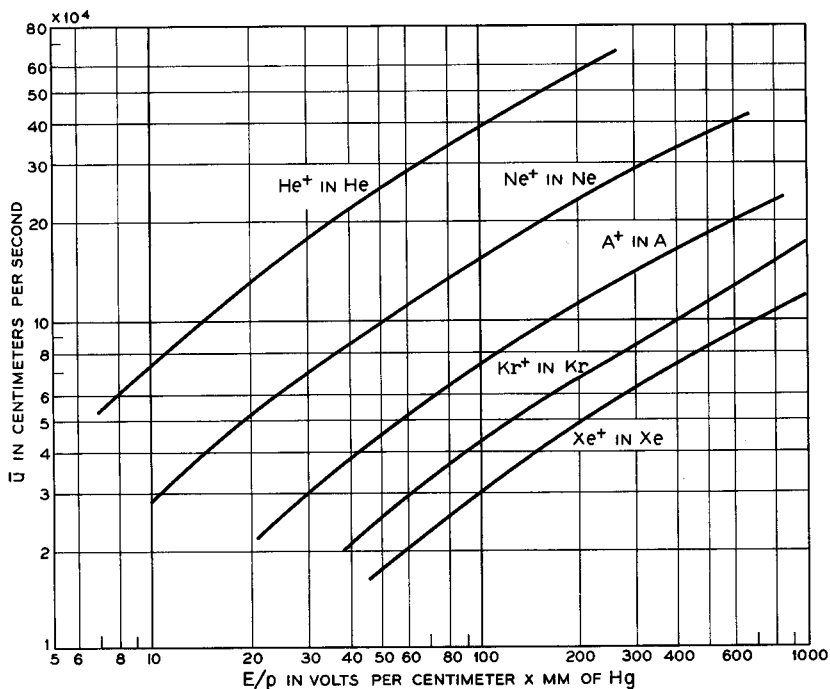


FIG. 14.3-1 Drift velocities of noble-gas ions through their parent gases as a function of  $E/p$ .

In Chapter 15 we shall find that the maximum value of  $E/p$  which is obtained in the normal glow discharge with molybdenum electrodes and neon gas is of the order of 200 volts/cm  $\times$  mm of Hg. At this value of  $E/p$  the drift velocity of neon ions in neon is about 2250 meters/sec, or approximately 4.2 times the thermal velocity of neon molecules. Since the thermal energy of a neon molecule is approximately 0.025 electron volt and since the lowest excited state of neon lies at 16.6 electron volts, there is essentially no excitation or ionization of neutral atoms by ions in the discharge. The same result applies to discharges in the other noble gases.

When free electrons drift through a noble gas under the influence of an electric field, the picture is considerably different. The concept that the drift velocity for weak electric fields is determined by a mobility is less meaningful for electrons because the electron mean free path is found to be a function of the electron energy. (It will be recalled that the expression for the ion mobility given by Equation (14.2-13) is directly proportional to the mean free path and that the mobility was assumed to be independent of ion

energy.) Figure 14.3-2 shows some experimentally measured<sup>9</sup> values of a quantity  $P_c$  proportional to the reciprocal of the mean free path of free electrons in the noble gases as a function of the square root of the electron energy. The abscissa is therefore proportional to the electron velocity.

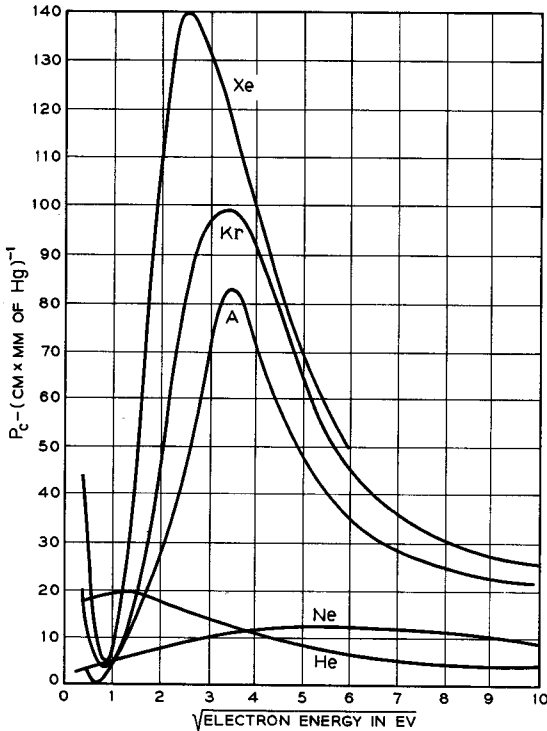


Fig. 14.3-2 The collision probability  $P_c$  plotted as a function of the square root of the electron energy for electrons in the noble gases. (From R. B. Brode, *Revs. Modern Phys.* 5, 257, 1933)

The quantity  $P_c$ , known as the collision probability, equals  $1/pL$ , where  $p$  is the gas pressure in millimeters of Hg and  $L$  is the electron mean free path in centimeters. In the case of the heavier gases,  $P_c$  drops to relatively small values at electron energies of about 1 ev. This effect, known as the Ramsauer-Townsend effect, can be explained quantum mechanically, and is discussed in several texts.<sup>10</sup>

<sup>9</sup>Reference 14.6.

<sup>10</sup>See, for instance, L. I. Schiff, *Quantum Mechanics*, McGraw-Hill Book Co., Inc., New York, 1949, p. 109.

For a given value of  $E/p$ , electrons gain much greater energies in drifting under the influence of the field than do ions. This effect can be attributed to the much smaller mass of the electrons. When an ion drifts through a gas under the influence of an electric field, the average energy transferred to the gas molecules per elastic collision is an appreciable part of the total kinetic energy of the ion. In a "head-on" elastic collision between an ion and a molecule in which the molecule is at rest and is of the same mass as the ion, the ion transfers all its kinetic energy to the struck particle. However, in a head-on elastic collision between an electron and a molecule, conservation of energy and momentum requires that the electron be reflected with almost all its kinetic energy. (The collision is in some ways similar to a collision between a ping pong ball and a bowling ball.) Consequently, even at relatively low values of  $E/p$ , of the order of 1 or 2 volts/cm  $\times$  mm of Hg, free electrons in neon gas may scatter their way up to sufficient energies to cause excitation of the gas molecules, the energy being gained as the electrons advance in the direction of greater potential. At higher values of  $E/p$  many of the electrons gain sufficient energy to cause ionization.

Table 14.3-1 lists some of the energies needed to excite and ionize the molecules of the noble gases. The metastable states referred to in the table are excited states for which the quantum-mechanical probability per unit time of direct transitions to the ground state by radiation is very small. An atom excited to such a state is frequently called a metastable.

TABLE 14.3-1. EXCITATION AND IONIZATION ENERGIES OF NOBLE GAS MOLECULES IN ELECTRON VOLTS

	$W_m$	$W_r$	$W_i$
He.....	19.80	21.21	24.58
Ne.....	16.62	16.85	21.58
A.....	11.55	11.61	15.77
Kr.....	9.91	10.02	14.01
Xe.....	8.32	8.45	12.14

$W_m$  = excitation energy of lowest metastable state.

$W_r$  = excitation energy of lowest level from which direct radiation to the ground state can take place.

$W_i$  = first ionization potential, or energy needed to remove a single electron from the outer shell of a neutral molecule.

When electrons make inelastic collisions with molecules of the noble gases in which excitation or ionization takes place, the electrons lose a large part of their kinetic energy. If a collision results in excitation of a neutral molecule, the kinetic energy lost by the electron equals the excitation energy of the molecule plus the kinetic energy gained by the molecule. Even

more energy is lost if ionization of a neutral molecule takes place. Consequently, the "steady-state" condition in which electrons drift through a gas under the influence of a moderate or large electric field is a dynamic condition in which the electrons scatter their way up to energies sufficient to cause excitation or ionization, whereupon they abruptly lose most of their energy and start over again.

Each time an electron ionizes a neutral molecule, an additional electron is released which is also accelerated by the field and which may cause further ionization. An "avalanche" therefore results in which a single electron falling through a sufficiently large potential drop gives rise to many electrons.

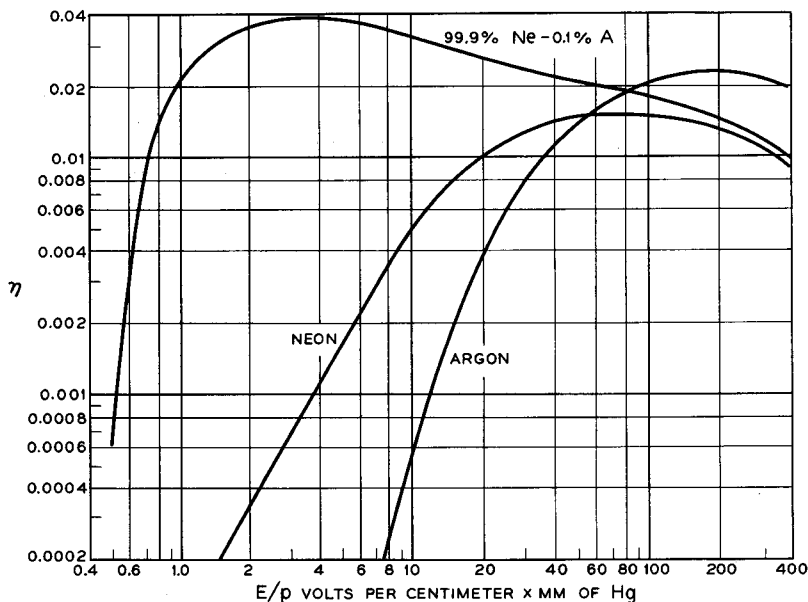


FIG. 14.3-3 The ionization coefficient  $\eta$  as a function of  $E/p$ . (From A. A. Kruithof and F. M. Penning, *Physica* 4, 430, 1937)

Figure 14.3-3 shows some measured<sup>11</sup> values of a quantity  $\eta$  known as the ionization coefficient per volt as a function of  $E/p$ .  $\eta$  is defined by the relation  $dN = \eta N dV$ , where  $dN$  is the incremental number of ion-electron pairs produced by  $N$  electrons in falling through  $dV$  volts. This definition implies that an "average" avalanche of  $e^{-\nu}$  electrons is generated by a single electron in falling through  $V$  volts. However, the values of  $\eta$  shown in Figure

<sup>11</sup>Reference 14.7.



14.3-3 apply only to electrons that have a velocity distribution characteristic of the "steady-state" condition in a region of uniform  $E/p$ . If the field is established between planar electrodes and if the electrons begin at the cathode ( $V = 0$ ) with relatively small energy, the size of the avalanche is more accurately expressed by  $e^{\eta(V-V_0)}$ , where  $V_0$  is a voltage which accounts for the potential drop through which the electrons must fall in order to acquire the energy distribution characteristic of the established field. Figure

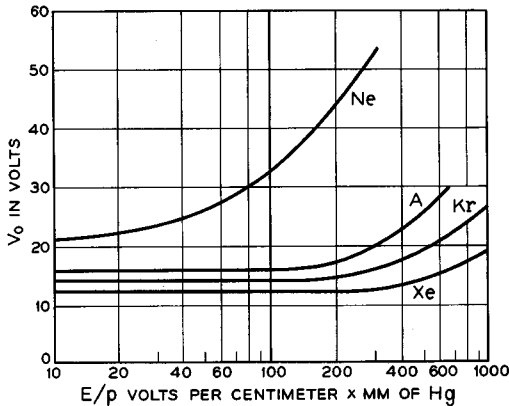


Fig. 14.3-4 The voltage  $V_0$  as a function of  $E/p$ . (From M. J. Druyvesteyn and F. M. Penning, *Revs. Modern Phys.* **12**, 87, 1940)

14.3-4 shows some measured values<sup>12</sup> of  $V_0$  as a function of  $E/p$  for several noble gases.

Of particular interest in Figure 14.3-3 is the curve for the gas mixture 99.9 per cent Ne — 0.1 per cent A since it is well above the curves for pure neon and pure argon over a considerable range of  $E/p$  values. The greater ionization in the Ne-A mixture results from a two-stage, modified  $\eta$  process in which neon metastables ionize neutral argon atoms. Reference to Table 14.3-1 shows that the lowest metastable state of neon is higher than the ionization potential of argon. Furthermore, it is known that when a neon atom is excited, the probability that it goes into a metastable state is high<sup>13</sup>, probably greater than 50 per cent for  $E/p$  less than 12 volts/cm  $\times$  mm of Hg. A current of fast electrons passing through the Ne-A mixture therefore generates a considerable number of neon metastables. The metastables diffuse through the gas with a relatively high probability of striking neutral argon atoms before being de-excited (see next section). Upon collision with

<sup>12</sup>Reference 14.8, p. 100.

<sup>13</sup>Reference 14.9, p. 463; Reference 14.8, p. 100.

an argon atom the excitation energy of the metastable is transferred to an outer-shell electron in the argon atom giving the electron sufficient energy to leave the atom. The neon metastable thereby becomes an unexcited atom, and an argon ion is generated. The phenomenon is known as the Penning effect.

For pure neon and argon the curves of  $\eta$  fall off at low values of  $E/p$  because of an increasing ratio of excitation to ionization and because of energy losses in elastic collisions. The curve of  $\eta$  for the Ne-A mixture falls off at low  $E/p$  because an increasing portion of the energy received by the electrons is lost in elastic collisions.

A measure of the voltage drop needed for a single electron to produce one more electron is obtained by setting  $e^{\eta V} = 2$ , or  $V = (1/\eta) \ln 2$ . For pure neon  $\eta$  has a maximum value of 0.015 at which  $(1/\eta) \ln 2 = 46$  volts. For pure argon  $\eta$  has a maximum value of 0.025 at which  $(1/\eta) \ln 2 = 28$  volts; and for the Ne-A mixture  $\eta$  has a maximum value of 0.037 at which  $(1/\eta) \ln 2 = 19$  volts. The latter value of  $\eta$  is the maximum value so far measured for any gas or gas mixture.

It is sometimes useful to define an additional ionization coefficient  $\alpha$  such that  $\alpha = \eta E$ .  $\alpha$  is known as the ionization coefficient per centimeter. In analogy to the definition of  $\eta$  given earlier,  $\alpha$  satisfies the relation  $dN = \alpha N dx$ , where  $dN$  is the number of ion-electron pairs produced by  $N$  electrons in traveling  $dx$  centimeters through the gas. In a discharge between planar electrodes in which there is a uniform electric field intensity between the electrodes, the electron current can be expressed as  $I = I_0 e^{\alpha(x-x_0)} =$

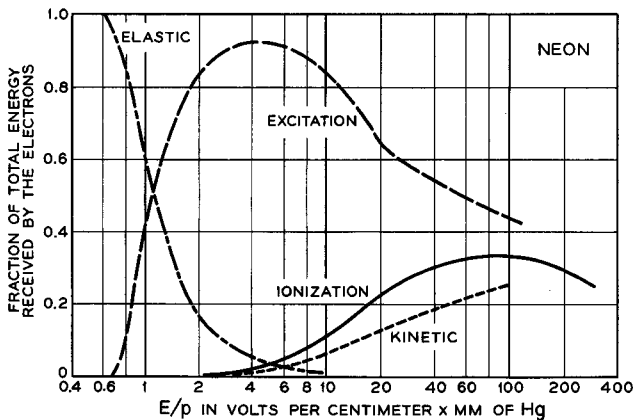


FIG. 14.3-5 The figure shows how the energy received by free electrons drifting through neon gas under the influence of a uniform electric field is spent as a function of  $E/p$ . From F. M. Penning, *Physica* 5, 286, 1938)

$I_0 e^{\eta(V-V_0)}$ , where  $x_0$  is the distance from the cathode at which the potential is  $V_0$ , and  $I_0$  is the electron current emitted from the cathode. Since  $\alpha = \eta E$ , and  $\eta$  is a function of  $E/p$ , the ratio  $\alpha/p$  is also a function of  $E/p$ .

Figure 14.3-5 shows how the total energy received by free electrons drifting through neon under the influence of an electric field is spent<sup>14</sup> as a function of  $E/p$ . Part of the energy received by the electrons is expended in elastic collisions (Elast.), part in inelastic collisions leading to excitation and ionization (Excit., Ion.), and part in accelerating newly released electrons up to the average energy (Kin.).

#### 14.4 Metastable Generation, Diffusion, and Destruction

Metastables play an important part in a number of gas-discharge phenomena. Metastables are generated by high-energy electrons striking

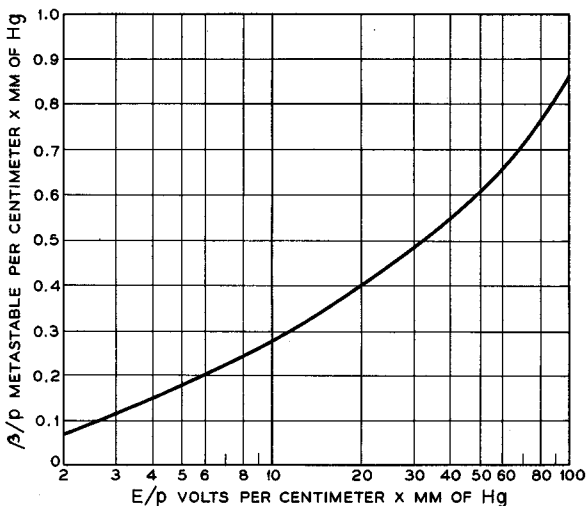


FIG. 14.4-1  $\beta$  measures the average number of metastables produced by an electron in advancing one centimeter in the direction of the field. The data are for free electrons in neon.

neutral molecules and by radiative transitions from other excited states. Since the metastables are uncharged, they diffuse through the gas and behave much like unexcited molecules (except that their effective diameter for collision is somewhat larger than that of the unexcited molecules).

<sup>14</sup>Reference 14.10, p. 288; Reference 14.8, p. 103.

Eventually each metastable experiences a collision which causes it to lose its identity as a metastable. In this section we shall present some data relating to metastable generation and diffusion, and the results of calculations based on the data. We shall also discuss briefly the mechanisms by which metastables are destroyed.

In analogy to the ionization coefficient  $\alpha$  we can define a coefficient  $\beta$  which expresses the average number of metastables produced by a single electron in advancing one centimeter in the direction of the field. Just as  $\alpha/p$  is a function of  $E/p$ , so also is  $\beta/p$ . Rogowski<sup>15</sup> has estimated  $\beta/\alpha$  for neon as a function of  $E/p$  by reducing Kruithof and Penning's<sup>16</sup> data for the ionization coefficient in neon-argon mixtures. His results are given in Table 14.4-1 together with Kruithof and Penning's values of  $\eta$ . The right-hand column in the table gives  $\beta/p = (\beta/\alpha)\eta E/p$ . Figure 14.4-1 shows a plot of  $\beta/p$  vs.  $E/p$ .

TABLE 14.4-1. IONIZATION AND METASTABLE GENERATION COEFFICIENTS FOR NEON

$E/p$ Volts/ cm $\times$ mm of Hg	$\beta/\alpha$ Metastables/ Ion-Electron Pairs (Ref. 14.11)	$\eta$ Ion-Electron Pairs/ Volt (Ref. 14.7)	$\beta/p$ Metastables/ cm $\times$ mm of Hg
2	102	0.00034	0.0694
5	22	0.0016	0.176
10	5.3	0.0050	0.265
20	2.1	0.0096	0.404
30	1.3	0.012	0.468
50	0.85	0.0142	0.604
100	0.6	0.0145	0.87

Metastables generated in a discharge of *low current density* are lost principally by (a) diffusion to the electrodes or walls of the containing vessel where de-excitation takes place, (b) collisions with neutral atoms leading to excitation or de-excitation to a nearby radiating state, from which a transition to the ground state can take place, and (c) collisions with two neutral atoms leading to the formation of an excited molecule which may be metastable.<sup>17</sup> In discharges of *higher current density* additional interactions be-

<sup>15</sup>Reference 14.11.

<sup>16</sup>Reference 14.7.

<sup>17</sup>Reference 14.12. In collisions leading to the formation of a metastable molecule, one of the neutral atoms becomes bound to the metastable, thus forming the molecule, and the other neutral atom carries away the binding energy of the molecule. The fact that the metastable molecule is stable, and the unexcited diatomic molecule is not, can be attributed to different electron configurations in the two cases.

tween metastables and metastables, between metastables and electrons, and between metastables and ions contribute to the destruction of the metastables. In the metastable-metastable interactions, one metastable becomes ionized and the other becomes de-excited.

In a discharge of low current density in which only the first three types of interaction are important, the equation expressing the density of metastables within the discharge is

$$\frac{\partial M}{\partial t} = D \frac{\partial^2 M}{\partial x^2} - ApM - Bp^2M + J(x)\beta(x)/e \quad (14.4-1)$$

where  $M$  is the number of metastables per unit volume,  $D$  is the metastable diffusion coefficient,  $p$  is the gas pressure in millimeter of Hg,  $A$  is the frequency of metastable destruction by two-body collisions at a pressure of 1 mm of Hg,  $B$  is the frequency of metastable destruction by three-body collisions at a pressure of 1 mm of Hg (both  $Ap$  and  $Bp^2$  have the dimensions (Time)<sup>-1</sup>),  $J(x)$  is the electron current density, and  $e$  is the electronic charge. It is assumed that the discharge takes place between planar parallel electrodes. The left-hand term in the equation and the first term on the right comprise the diffusion equation, Equation (14.2-6). The terms  $ApM$  and  $Bp^2M$  express the volume rate of destruction of metastables, and the term  $J(x)\beta(x)/e$  expresses the volume rate of generation of metastables. In the steady-state condition both sides of the equation are zero.

Phelps and Molnar<sup>18</sup> have measured the coefficients  $D$ ,  $A$ , and  $B$  for neon metastables in neon. They found that  $D = 146/p$  cm<sup>2</sup>/sec, where  $p$  is the pressure in millimeters of Hg,  $A = 50$  sec<sup>-1</sup> (mm of Hg)<sup>-1</sup>, and  $B = 0.5$  sec<sup>-1</sup> (mm of Hg)<sup>-2</sup>.

McClure<sup>19</sup> has used Equation (14.4-1) and the values of  $D$ ,  $A$ , and  $B$  given above to determine the steady-state current of metastable atoms reaching the cathodes of several devices having planar electrodes, a neon gas filling, and particular applied anode voltages. The current of metastables reaching the cathode was determined by calculating  $D\partial M/\partial x$  at  $x = 0$  using the boundary conditions that  $M = 0$  at  $x = 0$  and  $x = d$ , where  $d$  is the electrode spacing.

The results of McClure's calculations are shown in Table 14.4-2. The applied anode voltages  $V_a$  given in the table are the experimental breakdown voltages for devices with planar molybdenum electrodes and neon-gas fillings. The quantity  $G$  is the average number of metastables striking the cathode for each electron leaving the cathode. Later when we come to

<sup>18</sup>References 14.12 and 14.13.

<sup>19</sup>Reference 14.14.

TABLE 14.4-2. DATA RELATING TO AVALANCHES IN NEON GAS BETWEEN PLANAR ELECTRODES\*

$pd$ mm of Hg $\times$ cm	1.25	1.50	1.75	2.0	2.5	3	4	5	7	9	11
$V_a$ volts	148.5	146.0	145.0	145.1	147.6	151.9	162	173	196	220	239
$E/p$ volts/cm $\times$ mm of Hg	119	97	83	73	59	51	40	35	28	24	22
$e^{n(V_a - V_0)} = e^{n(d - x_0)}$	5.3	5.4	5.5	5.6	5.8	6.1	6.5	7.0	7.7	8.6	8.9
$G$ metastables/electron	0.62	0.67	0.71	0.75	0.79	0.81	0.80	0.74	0.62	0.52	0.43
$H$ metastables/electron	2.5	2.7	2.9	3.1	3.7	4.3	5.6	6.9	9.4	12	15
$K$ ions/electron	4.3	4.4	4.5	4.6	4.8	5.1	5.5	6.0	6.7	7.6	7.9

\*Pressure  $p = 40$  mm of Hg. The voltages  $V_a$  are the breakdown voltages for devices with planar molybdenum electrodes and neon-gas fillings.

discuss the breakdown phenomenon in Section 14.6 we shall make use of these results.

A second calculation of interest is the average number of metastables produced by the release of a single electron from the cathode. Assuming that few metastables are generated closer to the cathode than a distance  $x_o = (V_o/V_a)d$ , where  $V_o$  is the voltage plotted in Figure 14.3-4, this average number of metastables is easily shown to be

$$H = \beta \int_{x_o}^d \epsilon^{\alpha(x-x_o)} dx = \frac{\beta}{\alpha} (\epsilon^{\alpha(d-x_o)} - 1) \quad (14.4-2)$$

The ratio  $\beta/\alpha$  can be obtained by plotting a smooth curve of the data given in Table 14.4-1. Table 14.4-2 lists the values of  $H$  calculated in this manner for the particular applied voltages and  $pd$  values selected in the table. The quantity  $K$  given in the table is the average number of ions generated by each electron leaving the cathode.  $K$  is equal to  $\epsilon^{\alpha(d-x_o)} - 1$ .

#### 14.5 Emission of Electrons from Metal Surfaces by Ion, Metastable, and Photon Impact

The cold-cathode glow discharge is self-sustaining because products of the electron avalanches — ions, metastables, and photons — strike the cathode and release additional electrons. The self-sustaining condition is attained when the average number of discharge products generated by the release of a single electron from the cathode is sufficient to release one more electron. In normal glow discharges with noble gases and clean molybdenum or tungsten electrodes, about two thirds of the cathode emission results from ions striking the cathode, about one third results from incident metastables, and photoelectric emission may account for a few per cent.<sup>20</sup>

The emission of electrons from clean metal surfaces under impact by ions of the noble gases has been investigated in detail by Hagstrum.<sup>21</sup> The results of his investigations of the electrons emitted from molybdenum and tungsten by singly charged ions of the noble gases are shown in Figure 14.5-1. The quantity  $\gamma$  plotted in the figure is the average number of electrons emitted per incident ion. The measurements were taken using an exceptionally well evacuated system. Each observation was made immediately after the metal surface had been heated to a high temperature to drive off adsorbed layers of gas and before appreciable reabsorption had taken place.

<sup>20</sup>Reference 14.15.

<sup>21</sup>References 14.16 to 14.19.

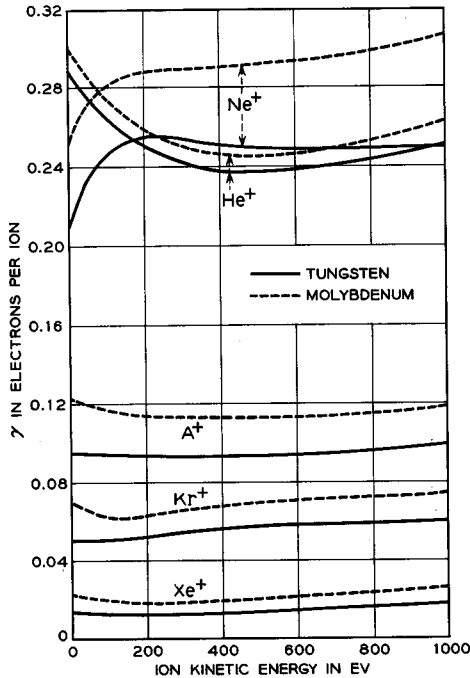


FIG. 14.5-1 The quantity  $\gamma$  for ions of the noble gases incident upon clean tungsten and molybdenum surfaces. (From H. D. Hagstrum, *Phys. Rev.* **104**, 672, 1956)

From his measurements Hagstrum concluded that for helium ions of kinetic energy less than 400 electron volts and for Ne, Ar, Kr, and Xe ions of kinetic energy less than 1000 electron volts the ion kinetic energy plays only a secondary role in the electron emission and is not directly responsible for the kinetic energy imparted to the emitted electrons. Instead, the emitted electrons derive their kinetic energy from the potential energy of the ions. The mechanism is as illustrated in Figure 14.5-2. As the ion approaches the metal surface, an interaction between the field of the ion and the fields of two electrons  $e_1$  and  $e_2$  in the conduction band of the metal causes one electron  $e_1$  to be captured by the ion, and the second electron  $e_2$  to be excited to an energy level well above the conduction band. The electron  $e_1$  goes into the lowest unfilled state of the ion making it an unexcited atom. This releases an amount of energy  $\epsilon'$  which is imparted to the second electron. Hagstrum concluded that the probability of this interaction taking place when noble-gas ions of 10 electron volts of kinetic energy or less are incident upon the metal surface is very nearly unity.



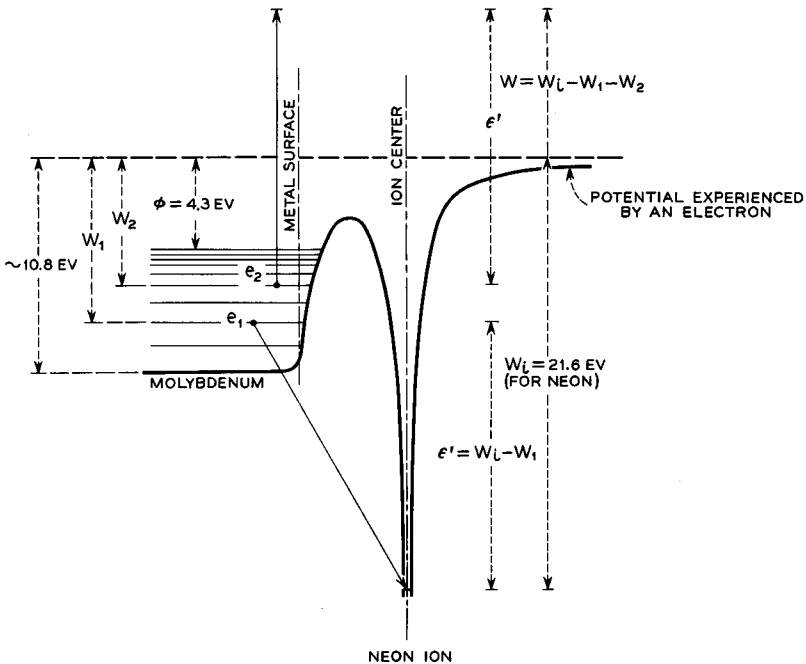


FIG. 14.5-2 The interaction that takes place at a molybdenum surface between an incident neon ion and two electrons in the conduction band of the metal.

The quantity  $\gamma$  therefore measures the probability that the electron  $e_2$  will escape from the metal when excited to the higher state.

Since the electrons  $e_1$  and  $e_2$  may initially have any energy within the conduction band, the excitation energy of the  $e_2$  electron just after the interaction may have a range of values. From Figure 14.5-2 it is easily shown that the maximum possible energy of the emitted electrons is  $W_i - 2\phi$ , where  $W_i$  is the ionization potential of the ion in electron volts, and  $\phi$  is the work function of the metal surface in electron volts. The emitted electron  $e_2$  has maximum energy only if both electrons  $e_1$  and  $e_2$  were originally in energy states at the top of the conduction band and if the electron  $e_2$  suffers no loss of energy before it leaves the surface. It is reasonable to expect that for different ions incident upon the same surface, high  $\gamma$  would be associated with a high value of  $W_i - 2\phi$ , and vice versa. From Figure 14.5-1 it is evident that, for low incident ion energies,  $\gamma$  decreases with increasing ion mass in going from He to Xe. This can be attributed to a corresponding decrease in  $W_i$  with increasing mass of the ions.

In investigating the emission of electrons from molybdenum by  $\text{He}^+$

ions Hagstrum found that the electron emission at low ion energies ( $\sim 10$  electron volts) was reduced by about a third when the surface was covered with a monolayer of helium gas. Varney<sup>22</sup> has measured  $\gamma$  for noble-gas ions striking a molybdenum surface through observations taken with a pulsed discharge. It is likely that the molybdenum cathode used in his measurements was covered with approximately a monolayer of noble gas, and hence more closely approximated conditions which are normally experienced in a gas-discharge device. Varney's measured values of  $\gamma$  are shown in Figure 14.5-3 as a function of  $E/p$ . The curves show nearly

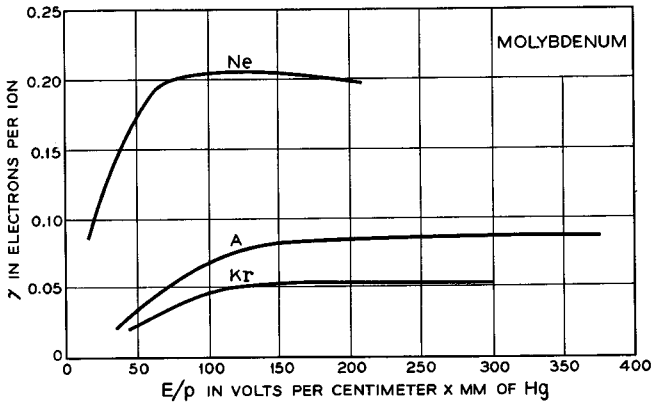


FIG. 14.5-3 Measurements of  $\gamma$  taken with a pulsed discharge. (From R. N. Varney, *Phys. Rev.* **93**, 1156, 1954)

constant values of  $\gamma$  for  $E/p$  greater than 150 volts/cm  $\times$  mm of Hg. These maximum values of  $\gamma$  are less than those observed by Hagstrum apparently because of the effects of adsorbed gas on the metal surface. For low values of  $E/p$  the curves become almost straight lines approximately directed at the origin. The reduction in  $\gamma$  at lower  $E/p$  is attributable to diffusion of the emitted electrons back to the surface as a result of collisions with gas molecules. As  $E/p$  increases, the probability of diffusion back to the cathode decreases; and for  $E/p$  greater than 150 volts/cm  $\times$  mm of Hg, virtually none of the emitted electrons return to the cathode.

Gases with polyatomic molecules, such as hydrogen, nitrogen, oxygen, and carbon dioxide, tend to become adsorbed on metal surfaces much more than the noble gases. When a tube is filled with a gas such as hydrogen, the cathode becomes covered with several monolayers, whereas a noble

<sup>22</sup>Reference 14.20.

gas may only cover the cathode with a single monolayer. As a result,  $\gamma$  for a discharge in hydrogen is very small, perhaps a few times  $10^{-4}$  at low  $E/p$ , where the ions have little energy with which to penetrate the adsorbed layers. At higher  $E/p$ ,  $\gamma$  takes on higher values, perhaps  $10^{-2}$  to  $10^{-1}$ , as the ions gain sufficient energy to approach the surface more closely.

Hagstrum concluded that, when a metastable atom of a noble gas is incident upon a clean molybdenum or tungsten surface, it experiences a two-stage interaction in which it first becomes ionized by losing its excited electron to the conduction band of the metal. The excited electron tunnels through<sup>23</sup> the potential barrier between the atom and the metal and occupies a vacant state in the metal directly opposite the metastable level. The incident particle is then indistinguishable from an incident ion so that an interaction similar to that illustrated in Figure 14.5-2 follows. It is therefore thought that the values of  $\gamma$  for metastable atoms of the noble gases incident upon clean molybdenum or tungsten surfaces are the same as for the corresponding ions.

The contribution made by photoelectric emission to the total cathode emission depends markedly on the work function of the cathode surface. Photoelectrons are emitted from the cathode because photons generated in the discharge strike the cathode and interact with electrons in the conduction band. The interaction is such that the energy of an individual photon is imparted to a single electron. The electron is energetically able to escape from the metal if it is excited to a state at least  $\phi$  volts above the conduction band, where  $\phi$  is the work function of the cathode surface. For a given energy  $h\nu$  of the incident photon, the maximum energy of the emitted photoelectrons is  $h\nu - \phi$ , where  $h$  is Planck's constant, and  $\nu$  is the frequency of the radiation. Low work function cathodes are in general more efficient emitters of photoelectrons, so that the photoelectron emission from an oxide-coated cathode (work function 1 to 2 electron volts) is likely to be much greater than that from a metal surface such as molybdenum (work function 4.3 electron volts). Because  $\gamma$  is quite small for discharges in gases with polyatomic molecules, the proportion of the total cathode emission arising from the photoelectric process is thought to be greater in these discharges than in discharges with the noble gases.

## 14.6 The Townsend Discharge and Breakdown

A gas discharge can be considered to be a state of conduction through a partially ionized gas between two electrodes. The current density of the

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<sup>23</sup>A quantum-mechanical effect in which an electron can pass through a potential barrier, even though classically the electron should not do so because it does not have sufficient kinetic energy to overcome the barrier.

discharge may vary over many orders of magnitude, from perhaps a few electrons per second per square centimeter ( $\sim 10^{-18}$  amp/cm<sup>2</sup>) to thousands or millions of amperes per square centimeter in the arc discharge. It happens that as the discharge current is increased above about 10 microamps, two independent phenomena start to show their influence on the discharge. Events which depend upon the square of the current density, such as metastable-metastable interactions and electron-metastable interactions, become of importance, and the space charge density in the interelectrode space starts to modify the potential distribution between the electrodes. A Townsend discharge<sup>24</sup> is characterized by a current density which is sufficiently small that neither of these effects are of importance.

In the present section we shall use the experimental data presented earlier to discuss the Townsend discharge and the phenomenon known as breakdown. Two discharges occurring at higher current densities, the glow discharge and the arc discharge, will be described in the first section of Chapter 15.

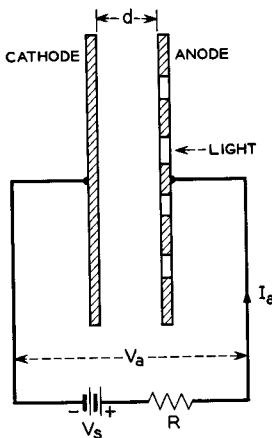


Fig. 14.6-1 A discharge device consisting of two large planar electrodes.

Let us suppose we have a device consisting of two large planar electrodes, a cathode and an anode, spaced by a distance of  $d$  centimeters, as illustrated in Figure 14.6-1. The interelectrode space is filled with a noble gas at a pressure of  $p$  millimeters of Hg. The applied voltage  $V_a$  is less than the breakdown voltage  $V_b$  of the device. The anode has a number of holes in it, and light is shone through the holes at the cathode. We shall assume that the incident light causes a current of  $I_0$  photoelectrons to be drawn from the cathode. Each photoelectron gives rise to an avalanche which in an average case consists of  $(\epsilon^n(V_a - V_0) - 1)$  ion-electron pairs

plus the original electron. The  $(\epsilon^n(V_a - V_0) - 1)$  ions generated in the avalanche are drawn to the cathode where they release  $\gamma(\epsilon^n(V_a - V_0) - 1)$  additional electrons. Metastables generated in the avalanches also strike the cathode and release electrons. If  $I_c$  is the total cathode emission, and  $G$  is

<sup>24</sup>After J. S. Townsend who contributed much to the early understanding of gas-discharge phenomena.

the average number of metastables striking the cathode as a result of the emission of a single electron from the cathode, then

$$I_c = I_o + \gamma(\epsilon^{\eta(V_a - V_o)} - 1 + G)I_c \quad (14.6-1)$$

or

$$I_c = \frac{I_o}{1 - \gamma(\epsilon^{\eta(V_a - V_o)} - 1 + G)} \quad (14.6-2)$$

where it is assumed that the contribution to  $I_c$  made by photons generated in the discharge is negligible. The current of electrons reaching the anode is  $I_c \epsilon^{\eta(V_a - V_o)}$ , or

$$I_a = \frac{I_o \epsilon^{\eta(V_a - V_o)}}{1 - \gamma(\epsilon^{\eta(V_a - V_o)} - 1 + G)} \quad (14.6-3)$$

This equation expresses the anode current of a Townsend discharge, when the discharge takes place between planar electrodes with a voltage  $V_a$  applied between the electrodes, and when a photoelectron current  $I_o$  is drawn from the cathode. The equation is known as the Townsend Equation.

As the applied voltage  $V_a$  increases, the denominator of the Townsend Equation decreases and the numerator increases.<sup>25</sup> At a particular applied voltage  $V_b$ , the denominator becomes zero and the current  $I_a$  may increase many orders of magnitude. The condition is known as breakdown, and the voltage  $V_b$  is called the breakdown voltage of the device.

The condition that the denominator of the Townsend Equation be zero is essentially the condition for a self-sustaining discharge, namely that the average number of discharge products generated by the release of a single electron from the cathode be sufficient to release one more electron from the cathode. In such a discharge the current  $I_a$  is no longer proportional to the photocurrent  $I_o$ , and in fact would continue if the light source were turned off. To limit the current of a self-sustained discharge, a resistance  $R$  is placed in series with the supply voltage. The voltage  $V_a$  is then  $V_s - I_a R$ , where  $V_s$  is the supply voltage.

Figure 14.6-2(c) shows qualitatively the variation of  $I_a$  with  $V_a$  for a particular device. The broken curve in the figure shows the  $I_a$ - $V_a$  characteristic for an increased photocurrent  $I_o'$ . The horizontal part of the solid curve in the Townsend region and the portions of the solid curve to the right of the Townsend region correspond to self-sustained discharges. Evidently, the current of the self-sustained discharge in the Townsend

<sup>25</sup>It should be noted that at low applied voltages, and hence at low  $E/p$ , many of the photoelectrons diffuse back to the cathode, so that increasing the applied voltage increases the current  $I_o$  of photoelectrons drawn from the cathode.

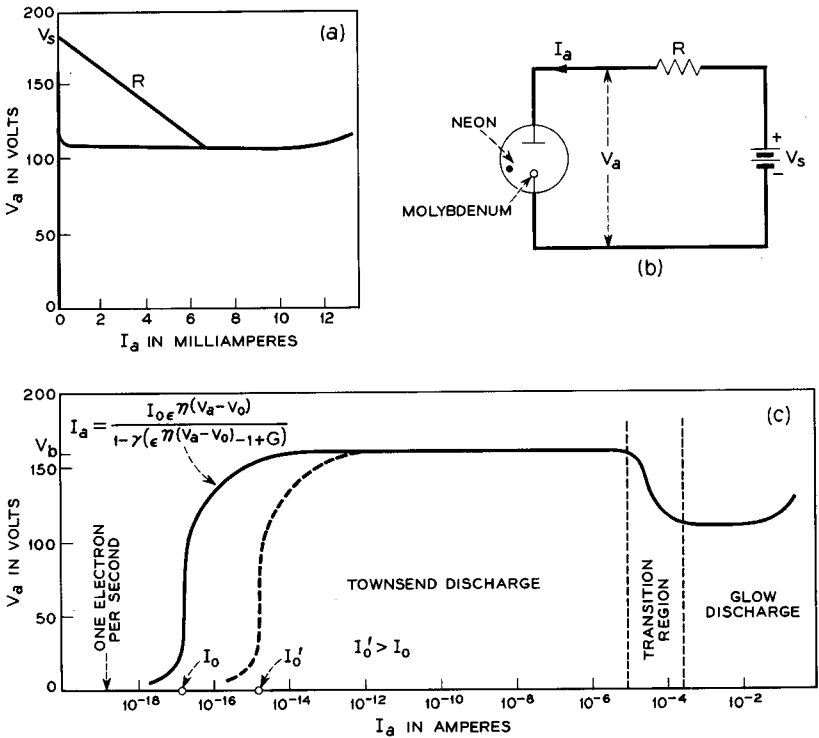


FIG. 14.6-2 Current voltage characteristics of a cold-cathode diode. (a) Current is plotted on a linear scale, and (c) current is plotted on a logarithmic scale.

region can be varied over several orders of magnitude with essentially no change in  $V_a$ .

To understand the stability of the self-sustained discharge we should note that the anode voltage of a self-sustained discharge is that voltage which makes the average avalanche contain enough discharge products to release one more electron from the cathode. Suppose that a discharge current  $I_a$  and an anode voltage  $V_a$  are obtained when the supply voltage is  $V_s$  and the series resistance is  $R$ . If the current  $I_a$  were to increase a small amount to  $I_a + \Delta I_a$ , the anode voltage  $V_a = V_s - I_a R$  would fall  $\Delta I_a R$  volts below the voltage needed for the discharge to be self-sustaining. The avalanches would then contain less than enough discharge products to release one more electron from the cathode, and the current  $I_a + \Delta I_a$  would decrease. The current  $I_a$  therefore adjusts itself to the value which makes  $V_a$  the voltage needed to sustain the discharge.

The condition for breakdown can be expressed as

$$\gamma(\epsilon^{\eta(V_b - V_o)} - 1 + G) = 1 \tag{14.6-4}$$

or

$$V_b = V_o + \frac{1}{\eta} \ln\left(\frac{1}{\gamma} + 1 - G\right) \tag{14.6-5}$$

where  $V_b$  is the breakdown voltage. Figure 14.6-3 shows a plot of  $V_b$  as a function of the product of the gas pressure  $p$  and the electrode spacing

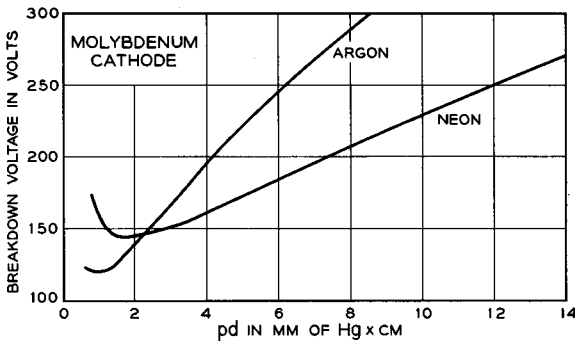


FIG. 14.6-3 The breakdown voltage for devices with planar molybdenum electrodes and neon and argon fillings.

$d$  for devices with planar molybdenum electrodes and neon and argon fillings. To a first approximation  $V_b$  is a function of the product  $pd$ , since at constant  $pd$  and constant applied voltage, both the average number of collisions made by an electron in traveling from the cathode to the anode and the average voltage gained in the interval between collisions are constant. The relationship that

$$V_b = f(pd) \tag{14.6-6}$$

is known as Paschen's Law.

It will be of interest to evaluate the quantity  $\gamma(\epsilon^{\eta(V_a - V_o)} - 1 + G)$  for one of the  $pd$  values shown in Table 14.4-2. The voltages  $V_a$  given in the table are the measured breakdown voltages for devices with planar molybdenum electrodes and neon fillings at a pressure of 40 mm of Hg. Consequently if we use the value of  $V_a$  indicated in the table and the value of  $\gamma$  for neon ions incident on molybdenum, the quantity  $\gamma(\epsilon^{\eta(V_a - V_o)} - 1 + G)$  should equal 1.0. The device we shall consider has a  $pd$  of 5 and hence an electrode spacing of 0.125 cm. The measured breakdown voltage is 173 volts. As the applied voltage is increased toward breakdown,  $E/p$  approaches

34.6 volts/cm  $\times$  mm of Hg. From Figure 14.5-3 we find that  $\gamma = 0.14$  for this value of  $E/p$ . From Table 14.4-2 the quantity  $\epsilon^{n(V_a - V_o)}$  is found to be 7.0 and  $G = 0.74$ . Hence  $\gamma(\epsilon^{n(V_a - V_o)} - 1 + G) = 0.94$ . The difference between 0.94 and 1.0 can easily be explained in terms of the accuracy of the experimental data we have used.

Since each value of  $pd$  defines a unique  $V_b$ , it also defines a unique  $E/p$ , namely  $V_b/pd$ . The breakdown voltage  $V_b$  can therefore also be plotted as a function of the value of  $E/p$  which applies at breakdown. Such a plot could be made from the data given in Table 14.4-2. Figure 14.6-3 shows that minimum breakdown voltage for a tube with a neon gas filling and planar molybdenum electrodes occurs at a  $pd$  of about 1.8 mm of Hg  $\times$  cm. At this  $pd$ ,  $V_b = 145$  volts, and  $E/p = 145/1.8 = 80$  volts/cm  $\times$  mm of Hg. Reference to Figures 14.3-3 and 14.5-3 shows that both  $\eta$  and  $\gamma$  are at their maximum values at approximately this value of  $E/p$ . Table 14.4-2 shows that  $G$  is a relatively slowly varying function of the value of  $E/p$  at breakdown. Consequently,  $V_b = V_o + (1/\eta) \ln [(1/\gamma) + 1 - G]$  is a minimum at  $E/p = 80$  volts/cm  $\times$  mm of Hg and hence at  $pd = 1.8$  mm of Hg  $\times$  cm.

When it is important that breakdown voltage of a cold-cathode tube be reproducible to within narrow limits, the pressure and electrode spacing are often chosen so that  $V_b$  is at its minimum value. In this case small variations in  $p$  and  $d$  from tube to tube make little change in the breakdown voltage.

Equation (14.6-6) is valid for electrode geometries other than the planar geometry if all electrode dimensions are scaled in proportion to the electrode spacing  $d$ .

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